注意：

允許學生個人、非營利性的圖書館或公立學校合理使用 IMC 各項試題及其解答。**可直接下載而不須申請**。

重版、系統地複製或大量重製本資料的任何部分，必須獲得 IMC 行政委員會的授權許可。

申請此項授權請電郵 IMC 行政委員會主席孫文先

ccmp@seed.net.tw

Notice:

Individual students, nonprofit libraries, or schools are permitted to make fair use of the papers and its solutions. Republication, systematic copying, or multiple reproduction of any part of this material is permitted only under license from the IMC Executive Board. Requests for such permission should be made by e-mailing Mr. Wen-Hsien SUN ccmp@seed.net.tw

International Mathematics Competition,

25~29 July, 2010, Incheon, Korea,

Invitational World Youth Mathematics Intercity Competition

Individual Contest

Instructions:

- Do not turn to the first page until you are told to do so.
- Remember to write down your team name, your name and contestant number in the spaces indicated on the first page.
- The Individual Contest is composed of two sections with a total of 120 points.
- Section A consists of 12 questions in which blanks are to be filled in and only **ARABIC NUMERAL** answers are required. For problems involving more than one answer, points are given only when **ALL** answers are correct. Each question is worth 5 points. There is no penalty for a wrong answer.
- Section B consists of 3 problems of a computational nature, and the solutions should include detailed explanations. Each problem is worth 20 points, and partial credit may be awarded.
- You have a total of 120 minutes to complete the competition.
- No calculator, calculating device, watches or electronic devices are allowed.
- Answers must be in pencil or in blue or black ball point pen.
- All papers shall be collected at the end of this test.

English Version

International Mathematics Competition,
25~29 July, 2010, Incheon, Korea,

Invitational World Youth Mathematics Intercity Competition

Individual Contest

Time limit: 120 minutes 27th July 2010 Incheon, Korea

Team: _____ Name: _____ No.: _____ Score: _____

Section A.

In this section, there are 12 questions. Fill in the correct answer in the space provided at the end of each question. Each correct answer is worth 5 points.

1. Real numbers p, q, r satisfy the equations $p+q+r=26$ and $\frac{1}{p}+\frac{1}{q}+\frac{1}{r}=31$. Find the value of $\frac{p}{q}+\frac{q}{r}+\frac{r}{p}+\frac{p}{r}+\frac{r}{q}+\frac{q}{p}$.

Answer : _____

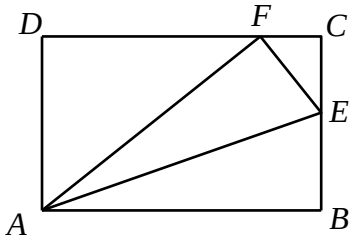
2. At a charity dinner, each person consumed half a plate of rice, a third of a plate of vegetables and a quarter of a plate of meat. Overall, 65 plates of food were served. What is the number of people at the charity dinner ?

Answer : _____

3. How many triples (x, y, z) of positive integers satisfy $xyz = 3^{2010}$ and $x \le y \le z < x + y$?

Answer : _____

4. E is a point on the side BC of a rectangle $ABCD$ such that if a fold is made along AE, as shown in the diagram below, the vertex B coincides with a point F on the side CD. If AD = 16 cm and BE = 10 cm, what is the length of AE, in cm ?



Answer : _____cm

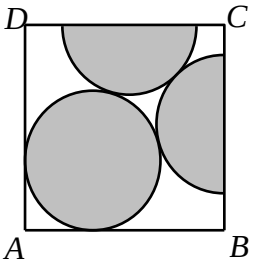
5. What is the smallest four-digit number which has exactly 14 positive divisors (including 1 and itself), such that the units digit of one of its prime divisors is 3?

Answer : _____

6. Let $f(x)$ be a fourth-degree polynomial. $f(t)$ stands for the value of this polynomial while $x=t$. If $f(1)=f(2)=f(3)=0$, $f(4)=6$, $f(5)=72$, what's the last digit of the value of $f(2010)$?

Answer : _____

7. A square *ABCD* circumscribed a circle and two semicircles each with radius 1 cm. As shown in the diagram, the circle and two semicircles touch each other, and two sides of the square touch the circle also. Find, in cm^2, the area of the square *ABCD*.



Answer: _____cm^2

8. Let p and q be prime numbers such that $p^3+q^3+1=p^2q^2$. What is the maximum value of $p+q$?

Answer : _____

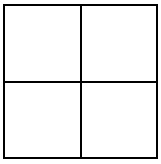
9. The sum of n positive integers, not necessarily distinct, is 100. The sum of any 7 of them is less than 15. What is the minimum value of n?

Answer : _____

10. P is a point inside triangle ABC such that $\angle ABP=20°$, $\angle PBC=10°$, $\angle ACP=20°$ and $\angle PCB=30°$. Determine $\angle CAP$, in degree.

Answer: _____°

11. A farmer has 100 pigs and 100 chickens. He has four yards each having square shape and forming together 2 × 2 grid. Farmer wants to distribute his animals into the yards in such way that first row has 120 heads, second row has 300 legs and first column has 100 heads, second column has 320 legs. How many different ways of doing this?



Answer : _____ways

12. An animal shelter consists of five cages in a row, labelled from left to right as shown in the diagram below. There is one animal in each cage.

Red Wolf	Silver Lion	Brown Fox	White Cow	Gray Horse

The five animals are indeed a wolf, a lion, a fox, a cow and a horse, and their colours are indeed red, silver, brown, white and gray. However, none of the labels matches any of the animals (For instance, the wolf is not red). Moreover, no animal is in or next to a cage whose label either matches its type or its colour. If the horse is not in the middle cage, what is the colour of the horse?
(Note：Write **R** for red, **S** for silver, **B** for Brown, **W** for white and **G** for Gray.)

Answer : _____

Section B.

Answer the following 3 questions, and show your detailed solution in the space provided after each question. Each question is worth 20 points.

1. Point *A* and *B* lie on the sides of a square, segment *AB* divides the square into two polygons each of which has an inscribed circle. One of the circles has radius 6 cm while the other one is larger. What is the difference, in cm, between the side length of the square and twice the length of segment *AB* ?

2. A small bag of candy contains 6 pieces. A medium bag of candy contains 9 pieces. A large bag of candy contains 20 pieces. If we buy candy in bags only, what is the largest number of pieces of candies which we cannot obtain exactly?

3. There is a list of numbers a_1, a_2, …, a_{2010}. For $1 \le n \le 2010$, where *n* is positive integer, let $S_n = a_1 + a_2 + \cdots + a_n$. If $a_1 = 2010$ and $S_n = n^2 a_n$ for all *n*, what is the value of a_{2010}?